CBCS SCHEME

USN | | | | | | | | | | | **18MAT11**

First Semester B.E. Degree Examination, June/July 2019
Calculus and Linear Algebra

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$. **(06 Marks)**

b. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts the x-axis. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

OR

2 a. Prove that the pedal equation of the curve $r^n = a^n\cos n\theta$ is $a^n.p = r^{n+1}$. **(06 Marks)**

b. Show that for the curve $r(1 - \cos\theta) = 2a$, ρ^2 varies as r^3. **(06 Marks)**

c. Find the angle between the polar curves $r = a(1 - \cos\theta)$ and $r = b(1 + \cos\theta)$. **(08 Marks)**

Module-2

3 a. Expand $\log(1 + \cos x)$ by Maclaurin's series up to the term containing x^4. **(06 Marks)**

b. Evaluate $\lim_{x \to 0}\left\{\frac{a^x + b^x + c^x}{3}\right\}^{1/x}$ **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

OR

4 a. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(06 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at the point (1, -1, 0). **(07 Marks)**

c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum. **(07 Marks)**

Module-3

5 a. Evaluate by changing the order of integration

$$\int_0^a \int_0^{2\sqrt{ax}} x^2 \cdot dy \cdot dx \ , \quad a > 0$$ **(06 Marks)**

b. Find the area bounded between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$. **(07 Marks)**

c. Prove that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m + n)}$ **(07 Marks)**

OR

6 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz.dy.dx$ **(06 Marks)**

b. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration. **(07 Marks)**

c. Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}.d\theta=\pi$ **(07 Marks)**

Module-4

7 a. Solve $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ **(06 Marks)**

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. **(07 Marks)**

c. Solve $yp^2+(x-y)\,p-x=0$. **(07 Marks)**

OR

8 a. Solve $\frac{dy}{dx}+y\cdot\tan x=y^3\cdot\sec x$ **(06 Marks)**

b. Find the orthogonal trajectory of the family of the curves $r^n\cdot\cos n\theta=a^n$, where a is a parameter. **(07 Marks)**

c. Solve the equation $(px-y)\cdot(py+x)=2p$ by reducing into Clairaut's form taking the substitution $X=x^2$, $Y=y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A=\begin{pmatrix}1&2&-2&3\\2&5&-4&6\\-1&-3&2&-2\\2&4&-1&6\end{pmatrix}$$ by applying elementary Row transformations. **(06 Marks)**

b. Solve the following system of equations by Gauss-Jordan method:
$x+y+z=9,\ 2x+y-z=0,\ 2x+5y+7z=52$ **(07 Marks)**

c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vector of the matrix $A=\begin{pmatrix}2&0&1\\0&2&0\\1&0&2\end{pmatrix}$ with $X^{(0)}=(1,0,0)^T$ as the initial eigen vector carry out 5 iterations. **(07 Marks)**

OR

10 a. For what values of λ and μ the system of equations.
$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ may have
i) Unique solution ii) Infinite number of solutions iii) No solution. **(06 Marks)**

b. Reduce the matrix $A=\begin{pmatrix}-1&3\\-2&4\end{pmatrix}$ into diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss-Seidel method
$20x+y-2z=17,\ 3x+20y-z=-18,\ 2x-3y+20z=25$. Carry out 3 iterations. **(07 Marks)**

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